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Exercise (Coordinate Plane and Distance Formula)

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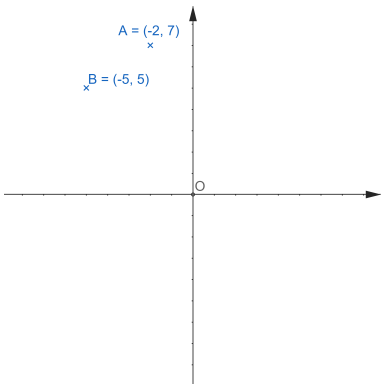
Part A: Basic Questions

1. In the figure, the coordinates of the points A ad B are $(-2, 7)$ and $(-5, 5)$ respectively. A is rotated clockwise about the origin 0 through $90°$ to A'. B' is the reflection image of B with respect to the y-axis.

   (a) Write down the coordinates of A' and B'.

   (b) Are the lengths of AB and $A'B'$ equal? Explain your answer.



2. The coordinates of the points P and Q are $(-3, 5)$ and $(2, -7)$ respectively. P is rotated anticlockwise about the origin O through $270°$ to P'. Q is translated leftwards by 21 units to Q'.

   (a) Write down the coordinates of P' and Q'.

   (b) Prove that PQ is perpendicular to $P'Q'$.

3. The rectangular coordinates of the point P are $(-1, \sqrt{3})$. If P is reflected with respect to the x-axis, find the polar coordinates of its image.

4. The point P is translated leftwards by 4 units to the point Q. If the coordinates of the reflection image of Q with respect to the y-axis are $(5, -1)$, find the polar coordinates of P.

5. If d is the distance between the point (a, b) and (b, a), then express d^2 in terms of a and b.

6. If $(-2, 3)$ is the mid-point of $(a, -1)$ and $(4, b)$, find the value of b.

Part B: Advanced Questions

7. If P is the point $(x, 0)$, Q the point $(0, 1)$ and R the point $(0, x)$, and $PQ = 2RQ$, find a quadratic polynomial with x as a root.

8. $A(-3, 2)$ and $B(1, 3)$ are two points. C is a point on the AB produced such that $AB : BC = 1 : 2$. Find the coordinates of C.

9. If the points $(0, 0)$, $(2, 0)$ and $(1, b)$ are the vertices of an equilateral triangle, then find the value of b.

10. The coordinates of the point A are $(-5, -2)$. A is translated rightwards by 9 units to the point B. B is then rotated anticlockwise about the origin through $90°$ to the point C. Find the y-coordinate of C.

11. If $\triangle ABC$ is an obtuse-angled triangle, which of the following points must lie outside $\triangle ABC$?

    (a) The centroid of $\triangle ABC$

    (b) The circumcentre of $\triangle ABC$

    (c) The orthocentre of $\triangle ABC$

12. Let O be the origin. The coordinates of the points P and Q are $(p, 0)$ and $(0, q)$ respectively, where p and q are positive numbers. If the in-centre of $\triangle OPQ$ lies on the straight line $3x + 4y = 3p$, find the value of $p : q$.

Solutions

1. (a) $A'(7, 2)$ and $B'(5, 5)$.

   (b) Note that $|AB| = \sqrt{(-2-(-5))^2 + (7-5)^2} = \sqrt{13}$ and $|A'B'| = \sqrt{(7-5)^2 + (2-5)^2} = \sqrt{13}$.

   Hence, they are equal.

   Also note that B' can be viewed as rotating B clockwise about the origin through $90°$, which will lead to the same conclusion.

2. (a) $P'(5, 3)$ and $Q'(-19, -7)$.

   (b) The slope of PQ is $\dfrac{-3-2}{5-(-7)} = \dfrac{-5}{12}$ and the slope of $P'Q'$ is $\dfrac{5-(-19)}{3-(-7)} = \dfrac{12}{5}$.

   The product is -1. Hence, PQ and $P'Q'$ are perpendicular.

3. The rectangular coordinates of the image are $(-1, -\sqrt{3})$.

   The first entry of the polar coordinates are given by $\sqrt{1^2 + (\sqrt{3})^2} = 2$.

   Note that the second entry θ satisfies $\cos\theta = -\dfrac{1}{2}$ and $\sin\theta = -\dfrac{\sqrt{3}}{2}$. Hence, $\theta = 210°$.

   Therefore, the polar coordinates are $(2, 210°)$.

4. The coordinates of Q is $(-5, -1)$ and Q is translated 4 units to the right to obtain the point P, which is $(-1, -1)$.

5. $d = \sqrt{(a-b)^2 + (b-a)^2}$.

   Hence, $d^2 = 2(a-b)^2$.

6. We only need $3 = \dfrac{-1+b}{2}$ to get $b = 7$.

7. Note that $|PQ| = \sqrt{1^2 + x^2} = \sqrt{1+x^2}$ and $|RQ| = |x-1|$.

   We have $\sqrt{x^2+1} = 2|x-1|$. Squaring both sides, we will get $3x^2 - 8x + 3 = 0$.

8. The straight line equation of AB is $x - 4y + 11 = 0$.

   Suppose the coordinates of C are $(4c-11, c)$.

   Note that $AB : BC = 1 : 2$, we know that $(c-2) : (3-c) = 1 : 2$. Hence, $c = \dfrac{7}{3}$.

   The coordinates of C is $(-\dfrac{5}{3}, \dfrac{7}{3})$.

9. Note that there are two equilateral triangles that both satisfy the question, above and below the x-axis respectively.

   $\sqrt{1^2 + b^2} = 2$

   Hence, $b = \sqrt{3}$ or $-\sqrt{3}$.

10. The coordinates of B are $(4, -2)$ and the coordinates of C are $(2, 4)$.

    Hence, the y-coordinate of C is 4.

11. Centroid is the concurrence of medians, hence it will always lie inside the triangle for all the triangles.

    Hence, circumcentre and orthocentre are the points which will lie outside the triangle for an obtuse angled triangle.

12. The in-center is the intersection of three angle bisectors of the triangle.

    Hence, it is on the straight line $y = x$.

    Note that it is also on the straight line $3x + 4y = 3p$.

We can write it in form of $(\frac{3}{7}p, \frac{3}{7}p)$.

The perpendicular distance between the in-center and PQ is also $\frac{3}{7}p$.

Hence, we have $\frac{|\frac{3}{7}pq + \frac{3}{7}p^2 - pq|}{\sqrt{p^2+q^2}} = \frac{3}{7}p$.

Then $p : q = 7 : 24$.