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# Exercise (Differentiation)

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## Part A: Basic Questions

1. Evaluate the following.

   (a) $\dfrac{d}{dx}(5x^6)$.

   (b) $\dfrac{d}{dx}(2x^{\frac{3}{2}})$.

   (c) $\dfrac{d}{dx}(4x^2 + 5x + 1)$.

2. Differentiate each of the following functions with respect to $x$.

   (a) $f(x) = 6x^{-\frac{3}{2}} + 4x + 1$.

   (b) $h(x) = 9x^2 - \dfrac{1}{2}x^4$.

   (c) $v(x) = \left(8x + \dfrac{1}{2}\right)^2$.

3. Differentiate the following expressions with respect to x.

   (a) $y = 2\sqrt{x} - 4\sqrt{x^3}$.

   (b) $y = x\sqrt{x} - \dfrac{1}{x^2}$.

   (c) $y = 4\sqrt{x} + \dfrac{1}{4\sqrt{x}}$.

4. For each of the following curves, find an equation of the tangent to the curve at the point whose $x$ coordinate is given.

   (a) $y = x^2 - 9x + 13$, where $x = 6$.

   (b) $y = x^4 + x + 1$, where $x = 1$.

   (c) $y = 3x^3 - 17x^3 + 24x - 9$, where $x = 2$.

5. For each of the following curves, find an equation of the normal to the curve at the point whose $x$ coordinate is given.

   (a) $f(x) = x^3 - 4x^2 + 1$, where $x = 2$.

   (b) $f(x) = x^3 - 7x^2 + 11x$, where $x = 3$.

   (c) $f(x) = \dfrac{1}{4}x^5 - 18x + 11$, where $x = 2$.

6. For each of the following equations, find the range of the values of $x$, for which $y$ is increasing or decreasing.

   (a) $y = 2x^3 - 3x^2 - 12x + 2$, increasing.

   (b) $y = x^3 - 6x^2 + 12$, decreasing.

**Part B: Advanced Questions**

7. The curve $C$ has equation

$$f(x) = 3x^2 - 8x + 2$$

   (a) Find the slope at the point on $C$, where $x = 1$.

   (b) The point $A$ lies on $C$ and the slope at $A$ is 4. Find the coordinates of $A$.

8. The curve $C$ has equation

$$y = -x^2(x+1)$$

   The curve meets the coordinate axes at the origin $O$ and at the point $A$.

   (a) Find the coordinates of $A$.

   (b) Show that the straight line with equation $x + y + 1 = 0$ is a tangent to $C$ at $A$.

9. The curve $C$ has equation

$$y = \frac{6}{x^2} + \frac{5x}{4} - 4, x \neq 0$$

   (a) Find an expression for $\frac{dy}{dx}$.

   (b) Determine an equation of the normal to the curve at the point where $x = 2$.

10. Let $y = x\sin x + \cos x$.

   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   (b) Let $k$ be a constant such that $x\frac{d^2y}{dx^2} + k\frac{dy}{dx} + xy = 0$ for all real values of $x$. Find the value of $k$.

11. Let $f(x) = e^x(\sin x + \cos x)$.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Find the value of $x$ such that $f''(x) - f'(x) + f(x) = 0$ for $0 \leq x \leq \pi$.

12. The curve $C$ has equation

$$y = \frac{x^3(5x\sqrt{x} - 128)}{\sqrt{x}}, x \in \mathbb{R}, x > 0$$

   (a) Determine expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

   (b) Suppose $A$ is a point on $C$ where the tangent to $C$ at $A$ has 0 slope. Show that the $y$-coordinate of $A$ is $-k\sqrt[3]{4}$, where $k$ is a positive integer.

   (c) Evaluate $\frac{d^2y}{dx^2}$ at $A$. Give the answer in terms of $\sqrt[3]{2}$.

   (d) Find the value of $\frac{d^3y}{dx^3}$ at the point on $C$, where $\frac{d^2y}{dx^2} = 0$.

**Solutions**

1. (a) $30x^5$.

   (b) $3x^{\frac{1}{2}}$.

   (c) $8x + 5$.

2. (a) $f'(x) = -9x^{-\frac{5}{2}} + 4$.

   (b) $h'(x) = 18x - 2x^3$.

   (c) $v'(x) = 128x + 8$.

3. (a) $\frac{dy}{dx} = x^{-\frac{1}{2}} - 6x^{\frac{1}{2}}$.

   (b) $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-3}$.

   (c) $\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{8}x^{-\frac{3}{2}}$.

4. (a) $\frac{dy}{dx} = 2x - 9$ will give us the slope of the tangent.
   Take $x = 6$ and we get 3.

   Also, the tangent need to pass the point $(6, -5)$.

   Therefore, the required tangent is $y = 3x - 23$.

   (b) $y = 5x - 2$.

   (c) $y = -8x + 11$.

5. (a) $\frac{dy}{dx} = 3x^2 - 8x$ will give us the slope of the tangent, which is the negative multiplicative reserve of the slope of the normal.

   Take $x = 2$ and we will get $-4$, hence the slope of the normal is $\frac{1}{4}$.

   Also, the normal should pass the point $(2, -7)$.

   Therefore, the required normal is $4y = x - 30$.

   (b) $4y = x - 15$.

   (c) $2y + x + 32 = 0$.

6. (a) $\frac{dy}{dx} = 6x^2 - 6x - 12$.

   When $\frac{dy}{dx}$ is positive, the original function is increasing.
   $6x^2 - 6x - 12 > 0$

   Hence, we have $x < -1$ or $x > 2$.

   (b) $0 < x < 4$.

7. (a) $\frac{dy}{dx} = 6x - 8$.
   Take $x = -1$, we get $-14$.

   (b) Suppose the $x$-coordinate of $A$ is $x$.

   We know that $x$ satisfies $6x - 8 = 4$.

   Therefore, $x = 2$ and the coordinates of $A$ is $(2, -2)$.

8. (a) On axes, we have either $x$ or $y$ is 0.

   Through direct computation, we can get the coordinates of $A$ is $(-1, 0)$.

(b) We compute the tangent to the curve at the point $A$.

$\frac{dy}{dx} = \frac{d}{dx}(-x^3 - x^2) = -3x^2 - 2x$.

Take $x = -1$ and we get the slope of the tangent is $-1$.

Also, the tangent should pass $A$, which exactly gives $x + y + 1 = 0$.

9. (a) $\frac{dy}{dx} = \frac{5}{4} - \frac{12}{x^3}$.

(b) $y = 4x - 8$.

10. (a) $\frac{dy}{dx} = x \cos x$.

$\frac{d^2y}{dx^2} = -x \sin x + \cos x$.

(b) Write out the equation explicitly, we get

$$x(-x \sin x + \cos x) + kx \cos x + x(x \sin x + \cos x) = 0$$

Compare the coefficient of $x$, $\sin x$, and $\cos x$, we will get $k = -2$.

11. (a) $f'(x) = 2e^x \cos x$.

$f''(x) = 2e^x(\cos x - \sin x)$.

(b) Write out the equation explicitly and compare the coefficient of $e^x \sin x$ and $e^x \cos x$, we will get $x = \frac{\pi}{4}$.

12. (a) $\frac{dy}{dx} = 20x^3 - 320x^{\frac{3}{2}}$.

$\frac{d^2y}{dx^2} = 60x^2 - 480x^{\frac{1}{2}}$.

$\frac{d^3y}{dx^3} = 120x - 240x^{-\frac{1}{2}}$.

(b) $x$-coordinate of $A$ is the solution to $\frac{dy}{dx} = 0$ with $x > 0$.

$x = 4\sqrt[3]{4}$.

Take $x$ into the original equation of $C$, and we get $k = 3072$.

(c) Take $x = 4\sqrt[3]{4}$ into $\frac{d^2y}{dx^2}$ and we get $960\sqrt[3]{2}$.

(d) Through direct computation, we get $x = 4$.

Take $x = 4$ into $\frac{d^3y}{dx^3}$ and we get 360.