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# Exercise (Equation of Straight Lines)

**Last updated: March 30, 2026**

## Part A: Basic Questions

1. If the lines $y = mx + b$ and $\dfrac{x}{a} + \dfrac{y}{b} = 1$ are perpendicular, find $m$ in terms of $a$ and $b$.

2. If the straight line $2x + y + k = 0$ passes through the point of intersection of the two straight lines $x + y - 3 = 0$ and $x - y + 1 = 0$, find the value of $k$.

3. The equation of the straight line $L$ is $kx + 4y - 2k = 0$, where $k$ is a constant. If $L$ is perpendicular to the straight line $6x - 9y + 4 = 0$. Find the $y$-intercept of $L$.

4. The equation of the straight line $L_1$ is $4x+3y-36 = 0$. The straight line $L_2$ is perpendicular to $L_1$ and intersects $L_1$ at a point lying on the $y$-axis. Find the area of the region bounded by $L_1$, $L_2$ and the $x$-axis.

5. $O$ is the origin. $A$ and $B$ are the points $(-2, 0)$ and $(4, 0)$ respectively. $l$ is a straight line through $A$ with slope 1. $C$ is a point on $l$ such that $CO = CB$.

   (a) Find the equation of $l$.

   (b) Find the coordinates of $C$.

   (c) Find the equation of the circle passing through $O$, $B$ and $C$.

   (d) If the circle $OBC$ cuts $l$ again at $D$, find the coordinates of $D$.

6. If the straight lines $hx + ky + 15 = 0$ and $4x + 3y - 5 = 0$ are perpendicular to each other and intersect at a point on the $x$-axis, then find $k$.

**Part B: Advanced Questions**

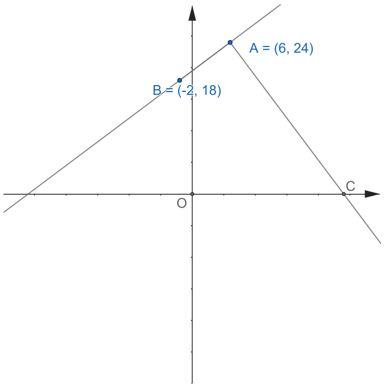
7. In the figure, the straight line passing through $A$ and $B$ is perpendicular to the straight line passing through $A$ and $C$, where $C$ is a point lying on the $x$-axis.

   (a) Find the equation of the straight line passing through $A$ and $B$.

   (b) Find the coordinates of $C$.

   (c) Find the area of $\triangle ABC$.

   (d) A straight line passing through $A$ cuts the line segment $BC$ at $D$ such that the area of $\triangle ABD$ is 90 square units. Let $BD : DC = r : 1$. Find the value of $r$.




8. The lines $3x - y - 8 = 0$ and $x - y - 2 = 0$ meet at a point $P$. $L_1$ and $L_2$ are lines passing through $P$ and having slopes $\frac{1}{2}$ and 2 respectively. Find their equations.

9. Let $J$ be the circle $x^2 + y^2 = r^2$, where $r > 0$.

   (a) Suppose that the straight line $L : y = mx + c$ is a tangent to $J$.

      i. Show that $c^2 = r^2(m^2 + 1)$.

      ii. If $L$ passes through a point $(h, k)$, show that $(k - mh)^2 = r^2(m^2 + 1)$.

   (b) $J$ is inscribed in a triangle $PQR$. The coordinates of $P$ and $R$ are $(7, 4)$ and $(-5, 5)$ respectively.

      i. Find the radius of $J$.

      ii. Using (a)(ii), or otherwise, find the slope of $PQ$.

      iii. Find the coordinates of $Q$.

10. Two straight lines $L_1 : x - 2y + 3 = 0$ and $L_2 : 2x - y - 1 = 0$. Find the equation of the straight line passing through $P$ and with equal positive intercepts, find the equation of $L$.

11. $A$ and $B$ are the points $(1, 2)$ and $(7, 4)$ respectively. $P$ is a point on the line segment $AB$ such that $\frac{AP}{PB} = k$.

   (a) Write down the coordinates of $P$ in terms of $k$.

   (b) Hence find the ratio in which the line $7x - 3y - 28 = 0$ divides the line segment $AB$.

12. The coordinates of the points $A$ and $B$ are $(5, 7)$ and $(13, 1)$ respectively. Let $P$ be a moving point in the rectangular coordinate plane such that $P$ is equidistant from $A$ and $B$. Denote the locus of $P$ by $\Gamma$.

   (a) Find the equation of $\Gamma$.

   (b) $\Gamma$ intersects the $x$-axis and the $y$-axis at $H$ and $K$ respectively. Denote the origin by $O$. Let $C$ be the circle which passes through $O$, $H$ and $K$. Someone claims that the circumference of $C$ exceeds 30. Is the claim correct? Explain your answer.

**Solutions**

1. $\dfrac{x}{a}+\dfrac{y}{b}=1$ can be changed into $y=-\dfrac{b}{a}x+b$.

   Since two lines are perpendicular, we have $m\cdot(-\dfrac{b}{a})=-1$.

   Hence, $m=\dfrac{a}{b}$.

2. The intersection of the two straight lines is $(1,2)$.

   Hence, $k=-4$.

3. Since two lines are perpendicular, we have $6k-36=0$.

   Hence, $k=6$.

   Then, the $y$-intercept of $L$ is 3.

4. The intersection on $y$-axis must be $(0,12)$.

   Note that $L_2$ is perpendicular to $L_1$, the equation of $L_2$ is given by $3x-4y+48=0$.

   The region bounded by $L_1$, $L_2$ and $x$-axis is a triangle with base $16+9=25$ and height 12.

   The desired area is 96.

5. (a) The equation of $l$ is $y=x+2$.

   (b) $C(2,4)$.

   (c) The $x$-coordinate of the center of the circle is $\dfrac{0+4}{2}=2$.
   Suppose the function is $(x-2)^2+(y-b)^2=r^2$ for some $b$ and $r$.
   Take $O(0,0)$ and $C(2,4)$ and we will get $b=\dfrac{3}{2}$ and $r=\dfrac{5}{2}$.
   Therefore, the equation of the circle is $(x-2)^2+(y-\dfrac{3}{2})^2=\dfrac{25}{4}$.

   (d) Take $y=x+2$ into the equation of the circle.
   We get $x=2$ or $-\dfrac{1}{2}$.
   Hence, the coordinates of $D$ is $(-\dfrac{1}{2},\dfrac{3}{2})$.

6. $(\dfrac{5}{4},0)$ is a point on the line $4x+3y-5=0$ and also on $x$-axis.

   Hence, it is a point on $hx+ky+15=0$. Therefore, $h=-12$.

   Since $hx+ky+15=0$ and $4x+3y-5=0$ are perpendicular, we have $4h+3k=0$.

   Therefore, $k=16$.

7. (a) The equation is $y=\dfrac{3}{4}x+\dfrac{39}{2}$.

   (b) The equation of $AC$ is $y=-\dfrac{4}{3}x+32$.
   Hence, the coordinates of $C$ is $(24,0)$.

   (c) Note that $|AB|=\sqrt{8^2+6^2}=10$ and $|AC|=\sqrt{18^2+24^2}=30$.
   Hence, the area is $\dfrac{10\cdot 30}{2}=150$.

   (d) $\dfrac{BD}{DC}=\dfrac{S_{\triangle ABD}}{S_{\triangle ADC}}=\dfrac{90}{150-90}=\dfrac{3}{2}$.
   Hence, $r=\dfrac{3}{2}$.

8. The coordinates of $P$ is $(3, 1)$.

   Hence, the equation of $L_1$ is given by $y = \frac{1}{2}(x-3)+1 = \frac{1}{2}x - \frac{1}{2}$.

   The equation of $L_2$ is given by $y = 2(x-3)+1 = 2x-5$.

9. (a) i. Combine the equation of $L$ and $J$, we have $x^2 + (mx+c)^2 = r^2$.
   Hence, $(1+m^2)x^2 + 2mcx + c^2 - r^2 = 0$.
   $\Delta = 4m^2c^2 - 4(1+m^2)(c^2-r^2) = 0$
   Therefore, $c^2 = r^2(m^2+1)$.

   ii. Put $(h, k)$ into $L$, we have $k = mh + c$.
   Hence, $(k-mh)^2 = c^2 = r^2(m^2+1)$.

   (b) i. The equation of $PR$ is given by $\frac{y-4}{x-7} = \frac{-5-4}{-5-7} = \frac{3}{4}$, which is $3x-4y-5=0$.
   Therefore, $x$-intercept$= \frac{5}{3}$ and $y$-intercept$= \frac{-5}{4}$.
   Hence, we have

$$\frac{1}{2}r\sqrt{(\frac{5}{3})^2 + (\frac{5}{4})^2} = \frac{1}{2}\cdot\frac{5}{3}\cdot\frac{5}{4}.$$

   Then, $r = 1$.

   ii. Use (a)(ii) with $(h, k) = (7, 4)$ and $r = 1$.
   We have $(4-7m)^2 = m^2+1$, which gives $m = \frac{3}{4}$ or $\frac{5}{12}$.
   Hence, $m_{PQ} = \frac{5}{12}$.

   iii. Use (a)(ii) with $(h, k) = R = (-5, 5)$ and $r = 1$.
   We have $(-5+5m)^2 = m^2+1$, which gives $m = \frac{3}{4}$ or $\frac{4}{3}$.
   Hence, $m_{QR} = \frac{4}{3}$.
   Let $Q = (a, b)$.
   We have $\frac{b-4}{a-7} = \frac{5}{12}$ and $\frac{b+5}{a+5} = \frac{4}{3}$.
   Solve $a$ and $b$, we have $Q = (\frac{-7}{11}, \frac{9}{11})$.

10. We first compute the coordinates of $P$, which is $(\frac{5}{3}, \frac{7}{3})$.

    Since $L$ has equal positive intercepts, its slope is $-1$.

    Hence, the equation of $L$ is $x+y-4=0$.

11. (a) The coordinates of $P$ is $(\frac{7k+1}{k+1}, \frac{4k+2}{k+1})$.

    (b) When $P$ lies on $7x-3y-28=0$,

$$7(\frac{7k+1}{k+1}) - 3(\frac{4k+2}{k+1}) - 28 = 0$$

    We get $k = 3$.

    Hence, the ratio is $3 : 1$.

12. (a) The equation of $\Gamma$ is:

    $(x-5)^2 + (y-7)^2 = (x-13)^2 + (y-1)^2$

    $4x - 3y - 24 = 0$.

    (b) $H(6, 0)$ and $K(0, -8)$

    Since $\angle HOK = 90°$, $HK$ is a diameter of $C$.

    Diameter$= \sqrt{6^2+8^2} = 10$

    Hence, the circumference of $C$ is $10\pi = 31.4 > 30$. The claim is correct.