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Exercise (Indices and Logarithms)

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Part A: Basic Questions

1. Which of the following is rational?

A. $\sqrt{12^3}$

B. $\sqrt{4} \times \sqrt{3}$

C. $\sqrt{8} \div \sqrt{2}$

D. $\sqrt{8} + \sqrt{8}$

E. $\sqrt{3} - \sqrt{2}$

2. Express $\sqrt{25a} - \sqrt{4a}$ in terms of a .

3. (a) Simplify $\frac{a^{\frac{5}{4}} \sqrt[4]{a^3}}{a^{-2}}$.

(b) Simplify $\frac{x^3 y^2}{x^{-3} y}$ and express your answer with positive indices.

(c) Simplify $\frac{a^3 a^2}{b^{-2}}$ and express your answer with positive indices.

4. (a) Evaluate $\log_2 8 + \log_2 \frac{1}{16}$.

(b) Simplify $\frac{\log a^3 b^2 - \log ab^2}{\log \sqrt{a}}$.

5. Let a and b be constants. Denote the graph of $y = a + \log_b x$ by G . The x -intercept of G is 9 and G passes through the point $(243, 3)$. Express x in terms of y .

6. Solve $4^x = 10 - 4^{x+1}$.

7. Solve the following equations without using a calculator:

(a) $3^x = \frac{1}{\sqrt{27}}$

(b) $\log x + 2 \log 4 = \log 48$

Part B: Advanced Questions

8. Let $x > y > 0$. If $\log(x + y) = a$ and $\log(x - y) = b$, then express $\log \sqrt{x^2 - y^2}$ in terms of a and b .
9. If a and b are positive integers, then express $\log_{a^b} b^a$ in terms of a and b .
10. Let $b > 1$. If $a = \log_{12} b$, then express $\frac{1}{a}$ in terms of b .
11. If $\frac{3}{3 \log x - 2} + 7 = \frac{2}{2 \log x + 1}$, then $\log \frac{1}{x} = ?$
12. If the roots of the equation $(\log_{\pi} x)^2 - 10 \log_{\pi} x + 24 = 0$ are α and β , then $\alpha\beta = ?$

Solutions

1. Ans: C.

A. $\sqrt{12^3} = \sqrt{2^6 3^3} = 24\sqrt{3}$ is irrational

B. $\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ is irrational

C. $\sqrt{8} \div \sqrt{2} = 2\sqrt{2} \div \sqrt{2} = 2$ is rational

D. $2\sqrt{8} = 4\sqrt{2}$ is irrational

E. Suppose $\sqrt{3} - \sqrt{2}$ was rational, then its square $5 - 2\sqrt{6}$ was rational, and thus $\sqrt{6}$ was rational, contradiction arises. Hence, $\sqrt{3} - \sqrt{2}$ is irrational.

2. Ans = $5\sqrt{a} - 2\sqrt{a} = 3\sqrt{a}$

3. (a) a^4

(b) $x^6 y$

(c) $a^7 b^2$

4. (a) $3 + (-4) = -1$

(b) $\frac{\log \frac{a^3 b^2}{ab^2}}{\log \sqrt{a}} = \frac{\log a^2}{\log \sqrt{a}} = \frac{4 \log \sqrt{a}}{\log \sqrt{a}} = 4$

5. We have

$$0 = a + \log_b 9 = a + 2 \log_b 3$$

$$3 = a + \log_b 243 = a + 5 \log_b 3$$

Hence, $a = -2, \log_b 3 = 1$.

$$y + 2 = y - a = \log_b x = (\log_b 3)(\log_3 x) = \log_3 x$$

Therefore, $x = 3^{y+2}$.

6. We have

$$4^x + 4^{x+1} = 10$$

$$5 \cdot 4^x = 10$$

$$4^x = 2$$

$$x = \frac{1}{2}$$

7. (a) We have

$$3^x = \frac{1}{\sqrt{27}} = \frac{1}{\sqrt{3^3}} = \frac{1}{3^{3/2}} = 3^{-3/2}$$
$$x = -\frac{3}{2}$$

(b) We have

$$\log x = \log 48 - 2 \log 4 = \log 48 - \log 4^2 = \log \frac{48}{4^2} = \log 3$$

Hence $x = 3$.

8. Ans = $\log \sqrt{x^2 - y^2} = \frac{1}{2} \log(x^2 - y^2) = \frac{1}{2} [\log(x + y) + \log(x - y)] = \frac{a + b}{2}$

9. Ans = $\frac{a \log b}{b \log a}$

10. Ans = $\frac{1}{\log_{12} b} = \log_b 12$

11. Ans = $-\frac{1}{2}$ or $\frac{1}{3}$
Let $t = \log x$, then we have

$$\frac{3}{3t-2} + 7 = \frac{2}{2t+1}$$

$$t = \frac{1}{2} \text{ or } -\frac{1}{3}$$

$$\log \frac{1}{x} = -t = -\frac{1}{2} \text{ or } \frac{1}{3}$$

12. Ans = π^{11}

Let $\log_{\pi} x = t$, then we have

$$t^2 - 11t + 24 = 0$$

$$\log_{\pi} \alpha\beta = \log_{\pi} \alpha + \log_{\pi} \beta = t_1 + t_2 = 11$$

$$\alpha\beta = \pi^{11}$$