

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
Exercises on Graph Theory

Question 1: If a vertex has 4 edges connected to it in an undirected graph, what is its degree?

Solution:

The degree is 4.

In an undirected graph, the degree of a vertex is simply the number of edges incident to (connected to) it.

Question 2: In an undirected graph with 6 vertices, can the maximum degree of a vertex be 6? Why or why not?

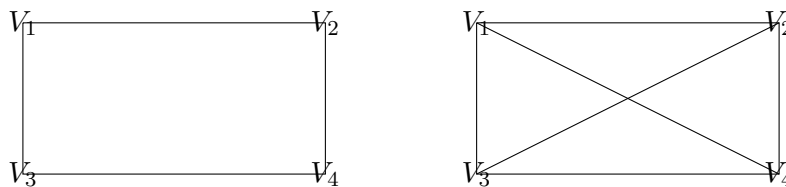
Solution:

No, it cannot be 6. The maximum possible degree is 5.

Why? A vertex cannot connect to itself (in a simple graph), and it can connect to at most all the other vertices. If there are 6 vertices total, one vertex can connect to at most the other 5 vertices.

So, maximum degree = $n - 1 = 5$.

Regular Graph: A graph in which every vertex x has the same/equal degree. A k -regular graph means every vertex has k edges. Every complete graph K_n is an $(n - 1)$ -regular graph, which means the degree is $n - 1$.



Regular Graphs

Question 3: For $V = \{1, 2, 3, 4\}$, find all unordered pairs and total edges in K_4 .

Solution:

Unordered pairs = $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$

Total = 6 edges. Formula check: $\frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$

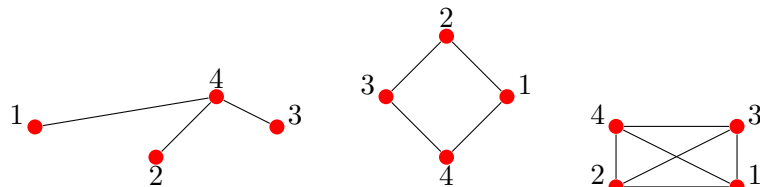
Question 4: Graph with edges $E = \{\{A, B\}, \{B, C\}, \{C, D\}, \{D, A\}\}$. Find the order and size of the graph and determine the degree of each vertex.

Solution:

This is a square cycle C_4 . Degrees: $\deg(A) = 2, \deg(B) = 2, \deg(C) = 2, \deg(D) = 2$

Question 5: Use the following undirected graphs to write three adjacency matrices accordingly.

The Adjacency Matrix of G is the square matrix where each v has a corresponding column and row. If two vertices v, u share an edge then there is a 1 in the corresponding location, otherwise a zero.



$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Question 6: Use the following adjacency matrix to draw an undirected graph.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$