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HSMMC Pre-workshop Exercise (Differential Equations)

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1. Solve the following separable differential equations.
   (a) $\dfrac{dy}{dx} = -4x^2y^3$;
   (b) $\dfrac{dy}{dx} = \dfrac{y^2+1}{xy}$ with $y(1) = 3$;
   (c) $\dfrac{dy}{dx} = xe^{y-x^2}$ with $y(1) = 0$.

2. Solve the first-order linear differential equation

$$y' + y = xe^{-x} + 1.$$

3. (Bonus) Consider the first-order linear differential equation

$$P(x)y' + Q(x)y = 0,$$

where $P(x)$, $Q(x)$ are given functions. Prove that if $y_1(x)$ and $y_2(x)$ are two solutions to the above equation, then for any constants c_1 and c_2, the function

$$y(x) = c_1y_1(x) + c_2y_2(x)$$

is also a solution.

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香港－上海中學生數學建模比賽 (HSMMC) 工作坊前練習（微分方程）

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1. 求解下列可分離微分方程。
   (a) $\dfrac{dy}{dx} = -4x^2y^3$；
   (b) $\dfrac{dy}{dx} = \dfrac{y^2+1}{xy}$，其中 $y(1) = 3$；
   (c) $\dfrac{dy}{dx} = xe^{y-x^2}$，其中 $y(1) = 0$。
2. 求解一階線性微分方程
$$y' + y = xe^{-x} + 1.$$
3. （挑戰題）考慮一階線性微分方程
$$P(x)y' + Q(x)y = 0,$$
其中 $P(x)$、$Q(x)$ 為已知函數。證明：若 $y_1(x)$ 和 $y_2(x)$ 是上述方程的兩個解，則對任意常數 c_1 和 c_2，函數
$$y(x) = c_1y_1(x) + c_2y_2(x)$$
亦是解。