

THE CHINESE UNIVERSITY OF HONG KONG

Department of Mathematics

Mathematical Modelling Project Team

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HSMMC Pre-workshop Exercise (Probability and Statistics)

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1. The probability that John passes a Math exam is $\frac{4}{5}$ and that he passes a Chemistry exam is $\frac{5}{6}$. If the probability that he passes both exams is $\frac{3}{4}$, find the probability that he will pass at least one exam.
2. A package of 6 light bulbs contains 2 defective bulbs. If 3 bulbs are selected for use, find the probability that none of the three is defective.
3. Three bits (0 or 1 digits) are transmitted over a noisy channel, so they will be flipped independently with probability 0.1 each. What is the probability that
 - (a) At least one bit is flipped?
 - (b) Exactly one bit is flipped?
4. It is known that:
 - The probability of having a rainy day is 0.1.
 - The probability that a person arrives at work late is 0.2.
 - The probability that the person arrives at work late given that the day is a rainy day is 0.8.

What is the probability that the day is a rainy day, given that the person arrives at work late?

5. Let X be a random variable having a probability mass function given in the following.

| | | | | | |
|------------|------|------|------|------|------|
| x | 0 | 1 | 2 | 3 | 4 |
| $P(X = x)$ | 0.52 | 0.28 | 0.14 | 0.04 | 0.02 |

Calculate the mean and variance of the random variable $Y = 4X + 3$.

6. (Bonus) Suppose that the error in the reaction temperature, in $^{\circ}\text{C}$, for a controlled laboratory experiment is a continuous random variable X having the density

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{for } -1 \leq x \leq 2, \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Verify that the probability indeed adds up to 1.
- (b) Find $P(0 < X < 1)$.

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香港－上海中學生數學建模比賽 (HSMMC) 工作坊前練習 (概率與統計)

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1. 約翰通過數學考試的概率為 $\frac{4}{5}$ ，通過化學考試的概率為 $\frac{5}{6}$ 。若他同時通過兩科考試的概率為 $\frac{3}{4}$ ，求他至少通過一科考試的概率。
2. 一包 6 個的燈泡中含有 2 個壞的。若隨機抽取 3 個燈泡使用，求選出的 3 個燈泡均為正常（非壞）的概率。
3. 三個比特（0 或 1 數字）在一個有雜訊的通道中傳輸，每個比特獨立翻轉的概率均為 0.1。求下列概率：
 - (a) 至少有一個比特被翻轉；
 - (b) 恰好有一個比特被翻轉。
4. 已知：
 - 下雨天的概率為 0.1。
 - 某人上班遲到的概率為 0.2。
 - 在雨天該人上班遲到的概率為 0.8。

求在該人上班遲到的情況下，當天為下雨天的概率。

5. 設 X 為一隨機變量，其概率質量函數如下表所示。

| | | | | | |
|------------|------|------|------|------|------|
| x | 0 | 1 | 2 | 3 | 4 |
| $P(X = x)$ | 0.52 | 0.28 | 0.14 | 0.04 | 0.02 |

計算隨機變量 $Y = 4X + 3$ 的期望值與方差。

6. (挑戰題) 設某對照實驗中反應溫度的誤差（以 $^{\circ}\text{C}$ 為單位）為一連續隨機變量 X ，其密度函數為

$$f(x) = \begin{cases} \frac{x^2}{3} & \text{當 } -1 \leq x \leq 2, \\ 0 & \text{其他情況。} \end{cases}$$

- (a) 驗證該概率密度函數的總概率確實為 1。
- (b) 求 $P(0 < X < 1)$ 。