Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		**2112262**

Wildfire monitoring mechanism based on the optimization model

Summary

For the wildfire that occurred in Victoria, this article designed a complete monitoring plan for Country Fire Authority (CFA). From daily monitoring to large-scale fire handling, this article gave a specific drone monitoring plan.

First, this article analyzes the effective shooting range and communication range of the surveillance and situational awareness drone (SSA) and repeater drone (RR), and determines the optimal flight route to reach the economic optimal solution. Then, this article analyzes the topographic characteristics of Victoria (mountains and sea level). According to different terrain features, this article gives different drones flying heights and arrays.

Next, this article made an Optimal Control Model for the number of drones purchased, which is balanced the relationship between economy and reliability.Through the method of dynamic planning, we calculated the approximate plan of drone procurement quantity through the program. The two solutions deal with daily surveillance and information transmission when a wildfire occurs.

Then, we uses FDDI parameters to characterize the severity of the fire, and uses QR dynamic adjustment to meet different needs in different periods. At the same time, we innovatively applied the step-wise cluster analysis method to the prediction of surface temperature to establish a prediction model.

Finally, we optimized the drones distribution queue to adapt to different heights of terrain to ensure the effectiveness of monitoring and the safety of drones.And we gave the detailed budget of the entire system to help CFA lobby the Council to obtain the budget.

Keywords:

Optimal Control Model; Dynamic Adjustment Model; SCA Method;

Contents

1 Introduction

1.1 Background

Australia's wildfire, which lasted for approximately half a year from July 2019 to January 2020, stormed the southeastern part of the country. Among the most stricken areas was Victoria, which witnessed heavy damage and casualties. In fact, there is a 'bushfire season' every summer in Australia. Normally, wildfires are caused by lightning strikes and accidental sparks, However, other occurrences are related to Australia's special climate, which makes the wildfires very distinctive from those in California, USA.
Australia's vegetation is mostly eucalyptus, whose leaves contain a notable amount of tree oil. After they fall off, they accumulate at the roots of the trees. Once the temperature rises slightly, they easily cause spontaneous combustion. All these signs have brought significant pressure to Australia's wildfire system, especially to Victoria's fire protection system. Therefore, comprehensive wildfire prevention and control work are extremely urgent for local residents and local environment.
Scientists also noticed that the burning of this kind of leaves produce dense smoke, which may cause thunderstorms and lighting, and furthermore, cause secondary fires under high temperature.[2]

(a) New south wales ranchers look on as lightning strikes and a wildfire rages in 2019 (from Victoria file footage)

(b) Rare summer thunderstorm sparks new wildfires in

1.2 Problem Restatement

We need to design a wildfire monitoring system that uses drones as a carrier when combined with firefighters' wearable devices. The system needs to have practical applications in southeastern Australia to have both monitoring and detection mechanisms for such wildfire disasters. At the same time, it is necessary to accurately design the distribution of drones to assess the size of the wildfire and maintain information transmission to help the Country Fire Authority (CFA) formulate a fire fighting strategy.

- What we **Have**:
    1. Complete topographic information of southeastern Australia;
    2. Capabilities, performances and parameters of Surveillance and Situational Awareness(SSA) drones and Radio Repeater(RR) drones.

- What we **Should Do**:

  1. **Choose a combination of two drones.** On the premise of achieving effective detection, we search for the best combination of SSA drones and RR drones to balance the financial situation and the effectiveness of the monitoring system;
  2. **Design SSA drone path.**Within the communication connection range of the RR drones, it is also connected with wearable devices on the ground to conduct effective communications;
  3. **Determine the process of wildfire evolution.**In order to adapt to the wildfires in the next decade, we need to first understand the evolution process of wildfires in Victoria;
  4. **Proposed RR drones deployment algorithm.** In order to make our strategy more universal, we need a optimizing algorithm to deal with wildfires of different sizes on different terrains.

1.3 Our Work

In our paper, we established four models to resolve the problem. To start with, we first considered the camera model of the SSA drone. Then, we combined it with the evolution process of wildfires, and designed the SSA drone path model. In order to achieve a comprehensive solution, we use the three models as multi-objective optimization models. In order to make our model more realistic when mirroring the wildfire scene, we added

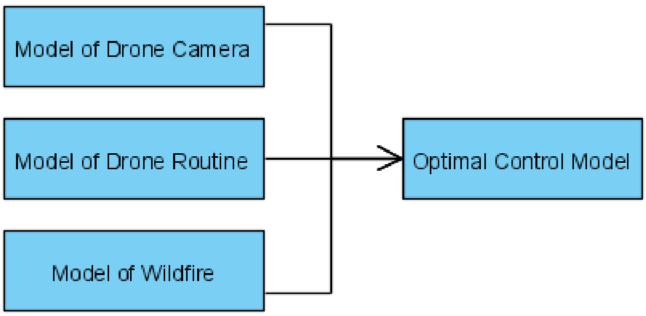


Figure 1: Strategy of Solving

the following factors to our model in particular:

- The influence of air resistance in high temperature air on the flight of drones;
- Limitation of the field of view (FOV) of the drone camera;
- The impact of different loads on the endurance of two drones.

Moreover, our paper discusses how to evaluate the effectiveness of the evaluations. In order to get more accurate results, we deliver special alarms and assessments for forests and near-forest cities.
Due to the uniqueness of Victoria, where the wildfires occur periodically, we innovatively

proposed the concept of **large and small years**. For different type of years, we offer different solutions implemented with different models respectively, and apply these models to a small island with similar topographic features as Victoria. These experiments show that our models are effective and universal.

2 Assumptions

2.1 Assumptions of Drones

- **Drones can be used repeatedly.**
- **Fixed cost of drone-related hardware facilities.** Fixed cost facilities include but are not limited to batteries, charging equipment, flight costs, aircraft losses, etc
- **Drones cannot operate without batteries.**This means that once a drone runs out of power, it has to return to the charging station to recharge before it starts to work again;
  This assumption is based on technical limitations to recharge the battery on site where fire outbreaks. Meanwhile, we assume both drones and supplementary communication equipment rely on electricity to operate.[3]
- **The connection between SSA drone and RR drone is only affected by temperature, which is negligible within a shorter period of time;**

2.2 Assumptions of Airborne Cameras

- **SSA drone's airborne cameras have stable model parameters.**We neglect the manufacturing differences of the cameras. Therefore, we expect stable quality of both the PTZ Cameras and the Static Cameras on the SSA drones.

2.3 Assumptions of Victoria Topography

- **The topological features of Victoria will not change significantly.** Over the times the eucalyptus forests have evolved to obtain great regeneration power after wildfires, thus the vegetation can quickly restore. Also, we ignore any alternation to the slope of the burning sites. This assumption ensures that the Forest Fire Danger Index solely depends on the temperature on the ground.
- **Ignore winds' disturbance on the drone body** The drone body has a small area of contact surface with the winds and a unique shape designed to avoid disturbance. Therefore, it cannot be easily swayed by winds and we only take winds' influence on the propellers of the drone.

3 Notation and Terminology

3.1 Terminology in Optimal Control Theory

Definition 3.1 (Controlled Dynamic)**.** Suppose that f depends on some "control" parameters belonging to a set $A \subset \mathbb{R}^m$; so that f : $\mathbb{R}^n \times A \to \mathbb{R}^n$. We select some value $a \in A$

and consider the corresponding dynamics:

$$\left\{\begin{array}{l}\dot{x}(t)=f(x(t), a) \quad(t>0) \\ x(0)=x^0\end{array}\right.$$

We obtain the evolution of our system when the parameter is constantly set to the value a.

Definition 3.2 (Control and Response)**.** We call a function $\alpha:[0, \infty) \rightarrow A$ a control. With respect to each control, we consider the ODE (ODE)

$$\left\{\begin{array}{l}\dot{x}(t)=f(x(t), \alpha(t)) \\ x(0)=x^0\end{array} \quad(t>0)\right.$$

and regard the trajectory $\mathrm{x}(\cdot)$ as the corresponding response of the system.

Definition 3.3 (Functional)**.** A Mapping f of an arbitrary set X into the set $\mathbb{R}$ of real numbers or the set $\mathbb{C}$ of complex numbers.
If X is endowed with the structure of a vector space, a topological space or an ordered set, then there arise the important classes of linear, continuous and monotone functional, respectively Linear functional; Continuous functional; Monotone mapping.

Definition 3.4.

$$\mathcal{A}=\{\alpha:[0, \infty) \rightarrow A \mid \alpha(\cdot) \text{ measurable }\}$$

Set A is used to denote the collection of all **admissible controls**, where

$$\alpha(t)=\left(\begin{array}{c}\alpha^1(t) \\ \vdots \\ \alpha^m(t)\end{array}\right)$$

.

Definition 3.5 (Optimal Control Theory in Functional Form)**.**

$$\min _{u(t) \in \Omega} J(u)=\varphi(\mathbf{x}(\mathrm{t}))+\int_{t_0}^t L(\mathbf{x}, \mathbf{u}) d t$$

Subject to $\mathbf{x}(t)=f(\mathrm{x}, \mathbf{u}, t), \quad x\left(t_0\right)=\mathbf{x}_0$ and $\Psi\left[x\left(t_0\right)\right]=0$
where $\Psi\left[x\left(t_0\right)\right]=0$ is called **constraint condition**,
$x\left(t_0\right)=\mathbf{x}_0$ is called **Terminal constraint**, and $J(u)$ is called **cost function**.
The phase portrait $\mathbf{u}$, which can minimize the cost function, is our target.

Under these circumstances,the number of drones we use is a positive integer. This indicates that our system is discrete, thus our cost function is

$$\min _{u(t) \in \Omega} J(u)=\varphi(\mathbf{x}(\mathrm{t}))+\sum_{t_0}^t L(\mathbf{x}, \mathbf{u}) \Delta t$$

3.2 Notation

Table 1: The List of Notation

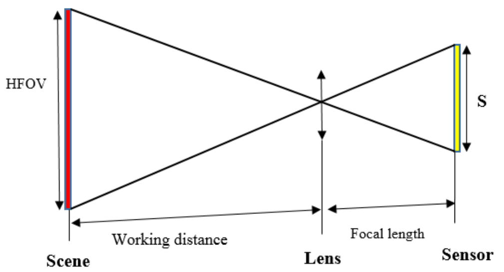
Symbol	Meaning
s_j	Area of j-SSA drone observable region
w_j	Area of j-RR drone communicable region
S	Area of the required observed region
$u_s(t)$	Number of SSA drones at t moment
$u_z(t)$	Number of RR drones at t moment
m_s	Cost of a SSA drone
m_j	Cost of a RR drone
M	Cost of a electric center for drone recharge
N	Cost of the control system

4 Model Preparations

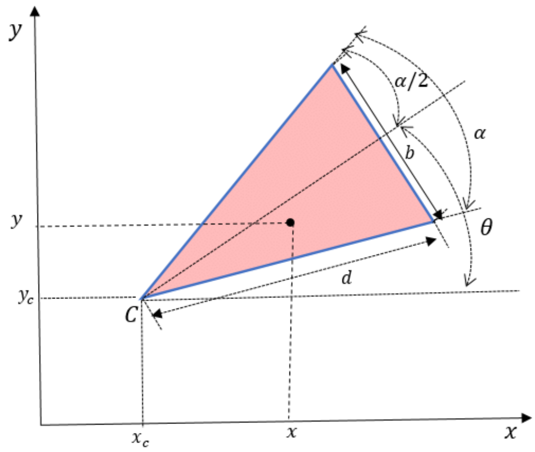
4.1 Detectable Range of the Airborne Camera

Oftentimes, the airborne camera of aerial drones with surveillance function and detection function is a combination of the PTZ camera and the Static Camera. In order to measure the quality of the pictures captured to obtain the effective detection distance of the SSA drone, we define some important parameters of aerial images here.[4]

Figure (a) illustrates the relationship between the fundamental parameters of the static camera, and Figure (b) illustrates the field of view of sensor C in a two-dimensional (2D) manner.



(a) Fundamental parameters of an imaging system



(b) Field of view of sensor C in 2D

In addition, this paper investigates the resolution of the PTZ camera with information presented below, which is obtained from the study by H. Bettahar et al. in 2014. [5] To

Table 2: Fundamental Parameters of an Imaging

Parameter	Potential damage
Field Of View (FOV)	The viewable area of the object under inspection. In other words, this is the portion of the object that fills the camera's sensor.
Working Distance (WD)	The distance from the front of the lens to the object under inspection.(vertical distance).
Resolution	The minimum feature size of the object under inspection.
Depth Of field (DOF)	The maximum object depth that can be maintained entirely in focus. DOF is also the amount of object movement (in and out of focus) allowable while maintaining a desired amount of focus.
Sensor Size	The size of a camera sensor's active area, typically specified in the horizontal dimension. This parameter is important in determining the proper lens magnification required to obtain a desired field of view.

achieve our goal of daily monitoring, the Emergency Operations Center (EOC) must meet the requirements of the detection zone.

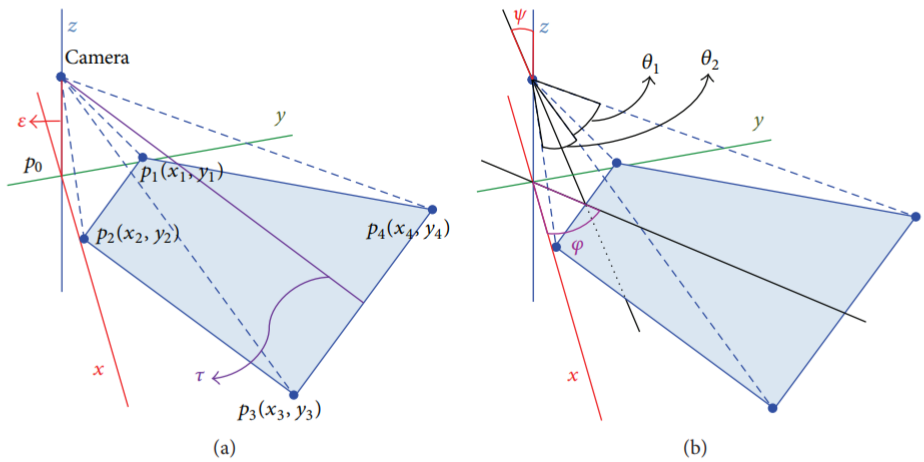


Figure 2: Surface-projected PTZ camera model based resolution requirement

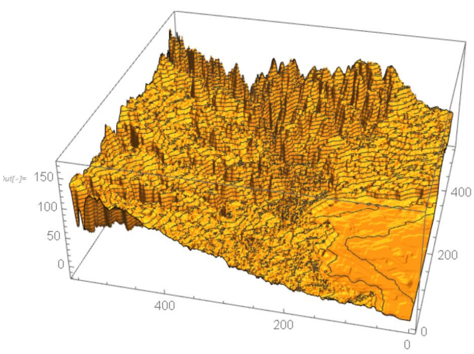
The final experimental results for PTZ cameras validates that the camera chosen only needs to be able to cover the detection zone to meet the detection requirements. Specific parameters are shown in the table below.

Table 3: Two Drones Parameters

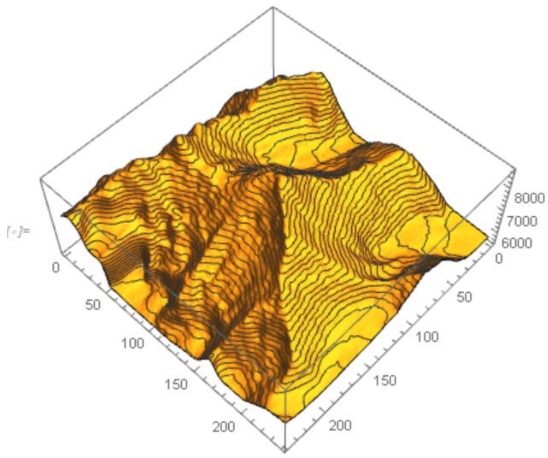
Parameters	SSA Drones	RR Drones
θ_1	$78°$	$78°$
Resolution	$1920 \times 1080(1080p)$	$1280 \times 720(720p)$
Cruising Distance	$4km$	$30km$
Cruising Duration	$0.5h$	$3.0h$
Communication Coverage Radius	$2.7km$	$30.0km$
Observation Coverage Radius	$5km$	No Need
Conventional Cruise WD	$120m$	Always same as SSA drones

4.2 Topographic Features of Victoria and Surrounding Regions

The entire Victoria State is very large, with an altitude ranging from 0 to 1986 meters above sea level at Bogong Mountain. According to the information given in the statements, the communication process of SSA is significantly affected by unfavorable topological features. SSA operates at its optimal when the signal transmission distance is between 2 to 5 kilometers. While the upper bound corresponds to the areas with few buildings or obstacles, the lower bound is responsible to explain the opposite scenarios. To make our models more persuasive, we choose two cities that match the descriptions. For the lower bound, we choose Buller, which is a mountainous area. For the upper bound, we choose Melbourne, which as an urban terrain and is well above the sea level.



(a) Melbourne (sea level urban terrain)



(b) Buller (mountain)

Here, the author selects Melbourne, a site with existing urban topography and sea-level topography, to study the monitoring strategy of coastal areas, and uses Buller, an area with a typical mountainous terrain to study the monitoring strategy of uneven terrains in inland areas.

It is worth noting that under different terrain conditions, SSA drones are affected by the radio communication range of wearable devices under different terrain conditions under'Boots-on-the-ground'. They need to ensure the communication coverage within the city. The communication coverage rate is 2 kilometers, and the communication coverage rate in a flat area is 5 kilometers.

4.3 Assessment of the Quantitative Severity of the Wildfire

This paper chooses the classic fire warning method proposed by Burgan et al. [5]. This method does not consider the external fire source, and uses the dynamic fire hazard Forest Fire Danger Index (FDDI) to predict the fire risk.

$$FDDI = 100 \times (1 - NDII7) \times \left(\frac{T - T_{\min}}{T_{\max} - T_{\min}}\right) \times (1 - P_{\nu})$$

where:

- P_{ν} is the percentage of vegetative cover, which is determined by NUVI, that is

$$P_{\nu} = \frac{NDVI - NDVI_s}{NDVI_v - NDVI_s}$$

  and define $NDVI_s = 0.15$ is the situation in completely bare soil and $NDVI_v$ is the situation in full vegetation cover;

- T is the predicted surface temperature;

- T_{min} and T_{max} are the maximum and minimum values of surface temperature in a period of time specifically.

- $NDII7$ is the normalized infrared index in the 7th band of MODIS, used to characterize the moisture content of vegetation.

Table 4: FFDI values for each fire danger rating class (Luke and McArthur 1986)

Fire Danger Rating	FFDI range
Low	0-5
Moderate	5-12
High	12-24
Very High	24-50
Extreme	50+

5 Model 1: Optimal Control Model

According to the previous analysis, for question 1, we need to solve this optimal control model in discrete version.

$$\min_{u(t)\in\Omega} J(u) = \varphi(\mathbf{x}(\mathrm{t})) + \sum_{t_0}^{t} L(\mathbf{x}, \mathbf{u})dt$$

$$\text{Subject to : } \mathbf{x}(t) = f(\mathrm{x}, \mathbf{u}, t), \quad x(t_0) = \mathbf{x}_0 \text{ and } \Psi[x(t_0)] = 0$$

The meaning of the constraint and cost function will be explained and modeled separately.

5.1 The Model of Constraint Condition

5.1.1 Communication constraint

The physical meaning of communication constraints is to ensure that each drone, no matter it is a SSA drone or a RR drone, can connect with the EOC within the specified region S that needs to be monitored. However, the SSA drone need to relay the signal through the RR drones because it solely has the ability to receive the signal. This limitation requires that, in a certain time period, the signal of the RR drones must cover the entire monitoring range so that the SSA drone can function.That is required that

$$u_z(t) \geq \frac{S}{w_j}$$

which is presented as

$$\Psi[u_z(t)] = u_z(t) - \frac{S}{w_j} \geq 0$$

Therefore,the communication constraint is $\Psi[u_z(t)] \geq 0$.

5.1.2 Monitoring Constraint

The physical meaning of monitoring constraint is to ensure that within the area S that needs to be monitored, every single spot is covered.

$$u_s(t) + u_z(t) \geq \frac{S}{w_j}$$

which is presented as

$$\Gamma[u_s(t), u_z(t)] = u_s(t) + u_z(t) - \frac{S}{w_j} \geq 0.$$

Therefore, the monitoring constraint is

$$\Gamma[u_s(t), u_z(t)] \geq 0.$$

From equations (1) and (2), the constraint condition for the Optimal Control Model is $\Gamma[u_s(t), u_z(t)] \geq 0$ and $\Psi[u_z(t)] \geq 0$.

5.2 The Model of Optimization Object

In the cost function, the key variables is the number of the SSA drones and RR drones. The **price function** is $\phi(\mathbf{u}(t))$:

$$\phi(\mathbf{u}(t)) = u_s(t) \times m_s + u_z(t) \times m_j + M + N.$$

where: m_s is the cost of a SSA drone; m_j is the cost of a RR drone; $u_s(t)$ is the number of SSA drones at t moment; $u_z(t)$ is the number of RR drones at t moment; M is the cost of a electric center for drone recharge; N is the cost of the drones control system.

At the same time, the system need to consider the **capability function** $L(u(t))$, which is reacted the area of the effective monitoring regions at t moment.

$$L(\mathbf{u}(t)) = s_j \times (u_s(t) + u_z(t))$$

However, due to the uncertainty of wildfire, its occurrence is uncertain, its spread range is uncertain, and its duration is also uncertain. To ensure that there are always SSA drones that can continue to monitor the risky area when a fire occurs, it is required that the number of SSA and RR drones should be greater than the number of drones that meet the monitoring constraints and communication constraints.

The general capability function $\chi(u(t)) = \int_{t_o}^{t} L(u(t))dt$ is more represented the real monitoring region, however, the number of the drones is not continuous, it's necessary to define a unit time t_P, and at this time line segment, the number of newly added two drones is $\Delta\mathbf{u}[u_s(t_p), u_z(t_p)]$. Therefore we have the discrete capability function

$$\chi(\mathbf{u}(t)) = \sum_{i=0}^{n} \sum_{t=t_0+i\times t_p}^{t_0+(i+1)\times t_p} L(\Delta\mathbf{u}(t)) = \sum_{t=t_0}^{t_0+T} L(\Delta\mathbf{u}(t)).$$

where defined the duration of the fire T.

Define the cost function by the linear quadratic regulator[6], that is

$$J(\mathbf{u}(t)) = Q \cdot \phi(\mathbf{u}(t)) - R \cdot \chi(\mathbf{u}(t))$$

Where Q is the economic index and R is the safety index.

By changing the proportions of Q and R,the demand for both can be adjusted. When it is predicted that extreme fire situations may occur, the proportion of R can be increased to increase safety; when no extreme fire situations are predicted,Q can be increased and lower economic cost.

5.3 The Solution of Optimal Control Model

In summary, the overall mathematical model of this optimization problem is

$$\min_{u(t)\in\Omega} J(\mathbf{u}(t)) = Q \cdot \phi(\mathbf{u}(t)) - R \cdot \chi(\mathbf{u}(t))$$

Subject to :$\Gamma\left[u_s(t), u_z(t)\right] \geq 0$ and $\Psi\left[u_z(t)\right] \geq 0$.

$$J(\mathbf{u}(t)) = Q \cdot \phi(\mathbf{u}(t)) - R \cdot \chi(\mathbf{u}(t)) \geq (2n-1) \times \max(\sum_{(i,j)\in A} \sum_{t\in T} \delta_{ij}^{t} x_{ij}^{t}) + \Delta t$$

$$= \sum_{(i,j)\in A} \sum_{t\in T} u_s(t)^t (f_{ij}^t + \sum_{(i,j)\in A} \mathrm{g}_{ij}^t) + M + N$$

Based on the above model and the parameters involved in the model, the final distribution plan is obtained by programming, and the result is shown in the table below:

Table 5: General Distribution Plan

Situation	SSA Drones	RR Drones
Routine Surveillance	252	79
Fire Response	413	117

Table 6: Routine Surveillance Distribution Plan

Class	Forest Region	High Risk Region
SSA Drones	179	73
RR Drones	47	32

Table 7: Fire Response Distribution Plan

Class	Forest Region	High Risk Region
SSA Drones	311	102
RR Drones	82	35

6 Model 2: Dynamic Adjustment Model

Question 2 indicates the possibility of extreme fire emergency in the next decade. The following shows how the model we set up in question 1 adapts the situation. Review Model 1: Optimal Control Model:

$$\min_{u(t)\in\Omega} J(u) = \varphi(\mathbf{x}(\mathrm{t})) + \sum_{t_0}^{t} L(\mathbf{x}, \mathbf{u}) dt$$

$$\text{Subject to : } \mathbf{x}(t) = f(\mathrm{x}, \mathbf{u}, t), \quad x(t_0) = \mathbf{x}_0 \text{ and } \Psi[x(t_0)] = 0$$

6.1 QR Dynamic Adjustment

Under normal circumstances, the area to be monitored S is a fixed value, the duration of the fire T is a regular value.
However, in case of severe fire emergency, the area to be monitored is larger than S, the duration of the fire is also larger than T. This paper use the following method to deal with this issue: changing the ratio of Q and R in order to adjust the demand in the real situation. When it is predicted that an extreme fire situation may occur, increase the proportion of R is to ensure safety; otherwise, improve Q and reduce economic costs.

The numbers of the two types of drones are in discrete form. Therefore, by altering the proportions of Q and R, the optimal total number of the two types of drones,

$$\min_{u(t)\in\Omega} J(u) = \varphi(\mathbf{x}(\mathrm{t})) + \sum_{t_0}^{t} L(\mathbf{x}, \mathbf{u})dt$$

implies that $min(u_s + u_z)$,which can be calculated when the loss function J is minimized. In that optimized case, we can get the total supervised area to be

$$S_{danger} = s_j \times min(u_s + u_z)$$

The fator of a fire risk is defined to be

$$K = \frac{S_{danger}}{S}$$

It takes the initial value of Q and R to both be 0.5, by fixing the$Q$ value and continuously adjusting the R value, the fire risk coefficient$K$ between different Q and R specific gravity can be obtained, as shown in the figure below

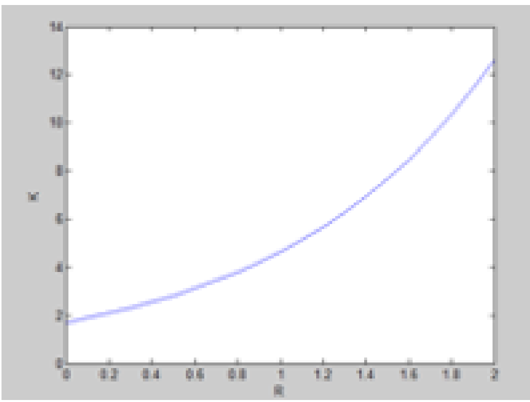


Figure 3: R-K Relation

As shown in the figure, when R is 0, the model only take economy in to consideration, in meeting the requirements of communication and monitoring, K = 1, $S_{danger} = S$, and the system only covers the given area to be monitored; but when R is 2, K is larger than 12, the actual monitored area is 12 times larger than the given area.

6.2 Stepwise cluster analysis for Annual Temperature

In section 4.3, this paper introduces an index to predict the wildfire.

$$FDDI = 100 \times (1 - NDII7) \times \left(\frac{T - T_{\min}}{T_{\max} - T_{\min}}\right) \times (1 - P_\nu)$$

where:

- P_ν is the percentage of vegetative cover, which is determined by NUVI, that is

$$P_\nu = \frac{NDVI - NDVI_s}{NDVI_v - NDVI_s}$$

  and define $NDVI_s = 0.15$ is the situation in completely bare soil and $NDVI_v$ is the situation in full vegetation cover;

- T is the predicted surface temperature;

- T_{min} and T_{max} are the maximum and minimum values of surface temperature in a period of time specifically.

- $NDII7$ is the normalized infrared index in the 7th band of MODIS, used to characterize the moisture content of vegetation.

In this evaluation model, the severe fire risk is affected in the following ways:

- The moisture content of combustibles can reasonably effect the possibility of forest fire;

- The vegetation coverage can be used as an auxiliary index to provide a corresponding reference;

- Surface Temperature affects the moisture content of combustibles, so the surface temperature can be used as one of the dynamic factors to predict the occurrence of forest fires.

6.3 SCA Method

The paper assumes that within ten years, there is no major change in the topography of Victoria and the vegetation coverage rate remains unchanged, so the fire risk index is only determined by the surface temperature.

The annual temperature prediction method used in this paper is the most used stepwise cluster analysis (SCA) in astronomical analysis. SCA is one of the non-linear methods based on data statistics. The advantage is that it can present a cluster tree to represent predictors and the complex interaction between predictors.

The SCA method was first cited by Liu and Wang M. to solve the multivariate modeling problem in medical research[7]. This is an innovation that using SCA method to predict the local temperature in Victoria.

This study uses the SCA model for stepwise cluster analysis, which uses the theory of multivariate analysis of variance (MANOVA) to help determine whether the difference between the two groups of dependent variables is significant at a given level of significance. The main method of this model is to perform a series of cutting and merging operations. Specifically, the model stipulates certain cutting and merging rules. When the cluster cannot be cut, the merging of the cluster will be executed; when no cluster can be merged with another cluster, the cutting operation will be executed; step by step, when all further hypothesis of cutting or merging is rejected, the cluster tree can be derived. The standard for cutting or merging is based on Wilks's statistics, which is defined as follows:

$$\Lambda = \frac{|\mathbf{E}|}{|\mathbf{E}+\mathbf{F}|}$$

Then the E and F can be given by

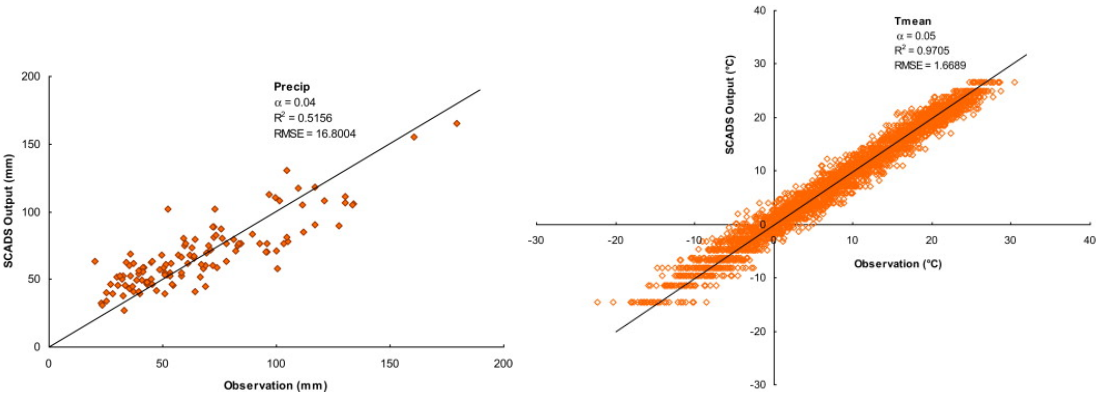
$$\mathbf{E} = \sum_{i=1}^{n_e} (\mathbf{e}_i - \overline{\mathbf{e}})' (\mathbf{e}_i - \overline{\mathbf{e}}) + \sum_{j=1}^{n_f} \left(\mathbf{f}_j - \overline{\mathbf{f}}\right)' \left(\mathbf{f}_j - \overline{\mathbf{f}}\right)$$

$$\mathbf{H} = \frac{n_e n_f}{n_e + n_f} (\overline{\mathbf{e}} - \overline{\mathbf{f}})' (\overline{\mathbf{e}} - \overline{\mathbf{f}})$$

where $\overline{e}$ is the sample mean of set $\mathbf{e}$, $\overline{\mathbf{f}}$ is the sample mean of set $\mathbf{f}$ respectively. They can be defined as follows:

$$\overline{\mathbf{e}} = \frac{1}{n_e} \sum_{i=1}^{n_e} \mathbf{e}_i$$

$$\overline{\mathbf{f}} = \frac{1}{n_f} \sum_{j=1}^{n_f} \mathbf{f}_i$$



(a) Validation for monthly precipitation at Victoria, 2010-2020.

(b) Validation for daily mean temperature at Victoria, 2010-2020.

According to the principles of statistics, the above statistics can be converted into expressions that can be F-tested, as shown below:

$$F\left(d, n_e + n_f - d - 1\right) = \frac{1-\Lambda}{\Lambda} \cdot \frac{n_e + n_f - d - 1}{d}$$

The F distribution can be used to test the above formula to determine whether there is a significant difference.

As shown in the figure below, we collected the meteorological data from 1980 to 2020 in Melbourne, Victoria. Through the above method, we can predict the temperature situation of the next ten years in Victoria, Australia.

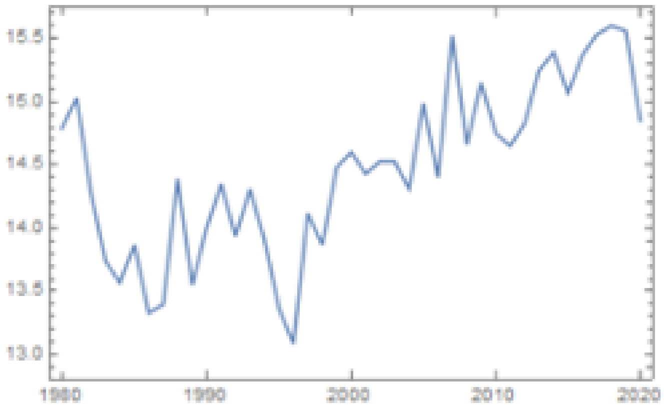


Figure 4: R-K Relation

7 Model 3: Optimize the Hovering Position of RR Drones

In the actual flight process, the drone will be disturbed by the wind field. The disturbance of the wind field will make the drone suffer from the wind force.

On the one hand, the wind field force increases the drag of the drones fuselage section; on the other hand, it changes the air-induced velocity under the rotor, which further changes the lift generated by the rotor. Considering the small cross-section of the drones fuselage and the air-permeable structure design of the fuselage, the influence of the wind field on the fuselage is ignored and only the effect of wind field forces on the rotor is considered. At different altitudes, different wind speeds will affect the cruise time of the drone and change the cruise distance. Therefore, the monitoring area and communication area are reduced.

According to the literature, if the wind speed is v_f, for a drones with a windward area b, the wind speed resistance is

$$F = \frac{b_f^2}{16}$$

. The time when it is needed to overcome the wind resistance is approximately calculated as h, then there is the cruise distance under the influence of wind speed

$$L_f = L - \frac{F \times H}{2M}$$

, where L is the calibrated cruise distance. Substituting L_f into the monitoring coverage model and the communication coverage model of question one, the wind resistance is different under different terrain conditions, so the annual average wind speed is approximated as the approximate wind speed of the place. The monitor coverage area, $s_{z,f}$(RR) $s_{s,f}$(SSA), and communication coverage area, $w_{z,f}$ (RR), $w_{s,f}$ (SSA) of two types of drones under the influence of wind resistance under different terrain conditions can be obtained respectively.

At the same time, the SSA drones under different terrain conditions, due to the effect of the ground personnel's handheld radio communication range under different terrain conditions, needs to ensure the communication coverage rate within the city, so the communication coverage rate is 2 kilometers. The regional communication coverage is 5 kilometers in plain. The communication coverage of SSA drones, $w_{s,d,f}$, under terrain conditions can be calculated according to different terrains. Then, by substituting different terrain and different parameters into the mathematical model established in Problem 1, the distribution of RR drones and SSA drones can be obtained under different terrain conditions.

8 Sensitivity Analysis

Review the model established in the previous section

$$\min_{u(t)\in\Omega} J(u) = \varphi(\mathbf{x}(\mathrm{t})) + \sum_{t_0}^{t} L(\mathbf{x}, \mathbf{u})dt$$

$$\text{Subject to : } \mathbf{x}(t) = f(\mathrm{x}, \mathbf{u}, t), \quad x(t_0) = \mathbf{x}_0 \text{ and } \Psi[x(t_0)] = 0$$

S is the area to be monitored, T is the duration of the fire, and Q and$R$ are parameters to measure economy and safety, respectively.

When answering the three questions of this problem, the above-mentioned optimal mathematical model is selected. Among them, the price of drone, cruise capability, monitoring coverage capability, communication coverage capability, fire monitoring area and time are all affected by the system and not affected by artificial settings. That is to say, the communication constraints $\Psi[x(t_0)] = 0$, monitoring constraints $\Gamma[u_s(t), u_z(t)] \geq 0$ are not affected by artificial settings. Only the economic coefficient Q and the safety coefficient R in the optimization objective function (loss function) are artificially set. To test the sensitivity of the model, we adjust the proportions of coefficients Q and R.

Same as the previous article, we choose Q as a fixed value of 0.5, and adjust R according to a step length of 0.1, and the adjustment interval is $[0, 1]$. We calculate in the interval$[0, 1]$ for each difference of 0.1 step, the SSA drones quantity change $\Delta\mathbf{u_s}(\mathbf{R})$, and the changes in the number of RR drones $\Delta\mathbf{u_z}(\mathbf{R})$. The results are shown below:

When R changes with a step of 0.1, the change of SSA drones $\Delta\mathbf{u_s}(\mathbf{R})$ is very large, which means a great impact on the number of SSAs. Change of the number of repeater drones is large in the early stage and getting smaller in the later period. However, the repeater drone mainly fulfill the communication coverage task and is expensive. The change in the number of repeater drones will cause the price changes drastically, so the quantity does not fluctuate much with the change of R.

Table 8:

R	$\Delta \mathbf{u_s}(\mathbf{R})$	$\Delta \mathbf{u_z}(\mathbf{R})$
0.1	89	15
0.2	98	15
0.3	109	14
0.4	121	14
0.5	134	14
0.6	147	2
0.7	163	0
0.8	180	2
0.9	199	1
1.0	211	2

In summary, the model selected in this question has high sensitivity. With the slight change in the economic factor Q and the safety factor R, the number of the two drones required will have a large change.

9 Strengths and Weaknesses

9.1 Strengths

This question selects the optimal method for modeling. The physical meaning of each parameter is clear, easy to explain, and can be calculated offline after the fire situation is predicted. When the fire danger situation is obtained, we can directly search the map to obtain the optimal number of drones to dispatch, greatly reducing the online calculation time. Compared with other nonlinear methods, model solving can obtain explicit solutions, instead of purely numerical solutions, which provides convenience for subsequent analysis.

9.2 Weaknesses and Extensions

The selected model for this question is the discretization form of the LQR model in the optimization method. In the modeling process, communication constraints and monitoring constraints are considered according to the meaning of the question, and wind resistance conditions are considered in the third question. However, in real engineering scenarios, after a forest fire occurs, the situation is complicated, such as flight inconvenience caused by heat waves and many other constraints. If more constraints are introduced, the complexity of the algorithm will be greatly increased and the dimensionality will explode. Bring solution difficulties, and even make it impossible to solve.

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To whom it may concern:

To minimize the lost due to the bushfire, we divide the problem in to two parts in our model, one is when everything is good, the other is when there is an emergency. We predicate the cost to be 5,575,000ASD to set up the project. This mainly includes buying equipment such as drones, developing and managing of the control system. Besides, to keep the system working, monitoring the state, for instance, it will cost 42,000 ASD per month. In case of an emergency, it will cost 370,000 ASD in extra to control and solve it.

According to our model, the optimized number of SSA drones is 413, which will cost 10,000 ASD. The hovering drones carrying repeaters will cost 1,170,000 ASD. To build electric center for drone recharging will cost 200,000 ASD. To develop a software control system, it will cost 75,000 ASD. So, the total amount to set up the project is 75,000 ASD.

When there is no emergency, we just need to make sure the drone system can cover the whole state. That requires 252 SSA drones, 79 repeater drones and 2 electric center. So, the cost per month need to cover the electricity fee, the basic repairing fee, the control system management fee. The estimated cost is 42,000 ASD per month.

In case of a fire emergency, the EOC will react accordingly, and the cost alter with respect to the fire level. The EOC will use the software to control the whole rescue operation. When it receives a fire signal from the drone system, it will deploy a mobile EOC near the fire location, in convenience of real situation management and supply. Also, EOC will send an emergency team, which will be directed by the newly sited mobile EOC latter to control the fire. To send a team will cost 120,000 ASD. What is more, it requires basic supplies to set up the mobile EOC, includes several drones to keep the whole firing area in surveillance, recharging device, equipment to physically control the fire situation. So, the actual cost will alter according to the level of severeness of the fire. Assume our model is sufficient, so that the scale of each fire is similar, then the cost to set up such a mobile EOC is 100,000 ASD. In summary, the total cost in case of an emergency will be 2,530,000 ASD.

In conclusion, the cost will include:

- 3,730,000ASD to set up the project;
- $42,000 \times 12$ for the system to work annually;
- 100,000ASD to settle one emergency. The request budget is 5,575,000ASD to for the system.